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Properties of General Pentagons

E 1990 [1967, 590]. *Proposed by V. F. Ivanoff, San Carlos, California*

Given a pentagon with the sides 1, 2, · · · , 5 in that order. Denoting by $p(q:r)$ the segment on the side p intercepted by the sides q and r, prove that

$$\text{(a)}\ \prod [1(3:5)] = \prod [5(1:3)], \qquad \text{(b)}\ \prod [1(4:5)] = \prod [5(1:2)],$$

where each factor is obtained by increasing the numbers of its predecessor by 1, the resulting numbers to be taken modulo 5. (Note: Both statements have been proved for cyclic pentagons. See W. B. Carver, *Cyclic Polygons*, this MONTHLY, 68 (1961) 537, with two illustrations.)

Solution by Stanley Rabinowitz, Far Rockaway, N.Y. The stated results, and others similar to them, are easily established by use of the law of sines. Let B be the intersection of sides 1 and 2 ($B=12$), $G=13$, $J=14$, $A=15$, $C=23$, $H=24$, $F=25$, $D=34$, $I=35$, $E=45$. Then

$$\text{(a)}\ \frac{\prod [1(3:5)]}{\prod [5(1:3)]} = \frac{AG}{AI}\cdot\frac{BH}{BJ}\cdot\frac{CI}{CF}\cdot\frac{DJ}{DG}\cdot\frac{EF}{EH}$$

$$= \frac{\sin \measuredangle I}{\sin \measuredangle G}\cdot\frac{\sin \measuredangle J}{\sin \measuredangle H}\cdot\frac{\sin \measuredangle F}{\sin \measuredangle I}\cdot\frac{\sin \measuredangle G}{\sin \measuredangle J}\cdot\frac{\sin \measuredangle H}{\sin \measuredangle F} = 1.$$

$$\text{(b)}\ \frac{\prod [1(4:5)]}{\prod [5(1:2)]} = \frac{AJ}{EJ}\cdot\frac{BF}{AF}\cdot\frac{CG}{BG}\cdot\frac{DH}{CH}\cdot\frac{EI}{DI}$$

$$= \frac{\sin \measuredangle E}{\sin \measuredangle A}\cdot\frac{\sin \measuredangle A}{\sin \measuredangle B}\cdot\frac{\sin \measuredangle B}{\sin \measuredangle C}\cdot\frac{\sin \measuredangle C}{\sin \measuredangle D}\cdot\frac{\sin \measuredangle D}{\sin \measuredangle E} = 1.$$