Confused Statements

370. [March, 1959] *Proposed by D. L. Silverman, Greenbelt, Maryland.*

Let xy denote x's statement to y. Determine the truth or falsity of the following set of statements:

AB: Someone is not lied to.
AC: Someone lies twice.
BA: Someone neither lies twice nor is lied to twice.
BC: Someone is lied to twice.
CA: Someone lies and is lied to.
CB: Someone does not lie.

Solution by Stanley Rabinowitz, Far Rockaway, New York.

Consider the truth-values of the statements AB and BC. There are only four possibilities (if these truth values exist):

(i) $AB \wedge BC$
       $\Rightarrow (\sim BA) \wedge (\sim CA)$ [since someone is lied to twice]
       $\Rightarrow BA$ [since B neither lies twice nor is lied to twice]
      contradiction

(ii) $AB \wedge (\sim BC)$
       $\Rightarrow AC$ [otherwise C would be lied to twice]
       $\Rightarrow CB$ [since A does not lie]
       $\Rightarrow (\sim BA)$ [since someone lies twice]
       $\Rightarrow BA$ [C neither lies twice nor is lied to twice]
      contradiction

(iii) $(\sim AB) \wedge BC$
       $\Rightarrow (\sim AC)$ [otherwise C would not be lied to]
       $\Rightarrow AC$ [A lies twice]
      contradiction

(iv) $(\sim AB) \wedge (\sim BC)$
       $\Rightarrow AC$ [otherwise C would be lied to twice]
       $\Rightarrow CB$ [otherwise B would be lied to twice]
       $\Rightarrow (\sim BA)$ [someone must lie twice]
       $\Rightarrow BA$ [C neither lies twice nor is lied to twice]
      contradiction

Since all these cases are inconsistent, the given set of statements must be self-contradictory.