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A Diophantine Cubic

580. [March, 1965] *Proposed by Joseph Arkin, Spring Valley, New York.*

Is a solution in integers possible for the equation $(c-a-b)^3=24\,abc$, where a, b and c are not zero?

Solution by Stanley Rabinowitz, Far Rockaway, New York.

I shall make use of the identity

$$24\,abc = (a+b+c)^3 - (a-b+c)^3 - (-a+b+c)^3 + (c-a-b)^3.$$

Substituting this in the given equation,

$$(c-a-b)^3 = 24\,abc,$$

gives

$$(a+b+c)^3 = (a-b+c)^3 + (-a+b+c)^3.$$

But it is known that the equation $x^3+y^3=z^3$ has no integral solutions unless x, y, or z is zero which would imply that a, b, or c were zero.

Hence, the given equation has no nontrivial integer solutions.