A Well-Known Derangement

667. [September, 1967] *Proposed by Lew Kowarski, Morgan State College, Maryland.*

In how many different ways can one place on a shelf N encyclopedia volumes so that no volume is in its proper place?

Solution by Stanley Rabinowitz, Far Rockaway, New York.

The number of ways is merely the number of derangements of n objects. This is a classic result and is known to be

$$!n = n \text{ subfactorial} = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right].$$

The problem has been proposed in many forms. I have found it in the following sources:

Ball and Coxeter, *Mathematical Recreations and Essays*, P. 46.
Chrystal, *Textbook of Algebra*, Part Two, P. 24.
Dorrie, *100 Great Problems of Elementary Mathematics*, Problem 6.
Dudeney, *Amusements in Mathematics*, Problem 267.
Graham, *Ingenious Mathematical Problems and Methods*, Problem 26.
Niven, *Mathematics of Choice*, P. 78.
Ryser, *Combinatorial Mathematics*, P. 22.

The problem is also equivalent to Problem 5 (Part I) of the Putnam Contest, November, 1958.